

## Convective Flow under Rotating Force

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## Abstract

A finite difference code using the primitive variables is used to simulate the mixed convection in air ( $Pr=0.7$ ) and liquid metals ( $Pr=0.015$ ). The present study involves numerical simulation of momentum and energy equations in order to analyze two dimensional mixed convection in air and liquid metals in a differentially heated square cavity subjected to rotation for a broad range of operating parameters i.e. Rayleigh number ( $Ra$ ), Taylor number ( $Ta$ ) and rotational Raleigh number ( $Ra_w$ ).

**Keywords:** Heat transfer, rotating force, fluid mechanics

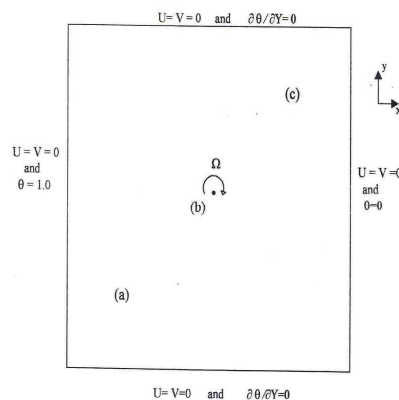
## Introduction

Natural convection in a differentially heated non-rotating enclosed cavity is encountered in various technological applications and has been studied extensively and the detailed flow and thermal structures are available in the literature. As the cavity is given rotation the flow in it is simultaneously affected by the Coriolis and Centrifugal forces as well as the thermal buoyancy. The resulting flow is expected to be rather complicated and is poorly understood so far. This finds wide applications in the study of geophysical flows and meteorology, ocean current and wind movement due to differential heating on the earth surface are some of the examples of the above mentioned phenomena. In the field of crystal growth the unit is rotated to stabilize the buoyancy induced flow so that the resulting crystal is of highest quality.

It is well known that a density gradient perpendicular to acceleration will produce motion no matter how small the gradient may be. The most common example being a fluid motion in a gravitational field when heated differentially. If this system undergoes rotation the centrifugal force can play a role analogous to that of gravity in producing motion. Rotation makes the flow complex and can exhibit profound influences on the convection when the different forces viz. Coriolis, Centrifugal and thermal buoyancy are of comparable magnitude. Rotation has been found to stabilize the fluid layers at large thermal gradient and subsequently delay convection when the rotation axis is aligned with the gravitational acceleration. However the role of rotation cannot always be to inhibit convection as it has been found that heat transfer is enhanced in some intermediate range of rotation rates.

**Mathematical Formulation:** Fig shows the Schematic diagram of the problem with boundary conditions. Initially at time  $t < 0$ , the air inside the enclosure is quiescent and isothermal at  $T_0$ . At  $t \geq 0$  the cavity is rotated at a constant angular speed  $\Omega$  about an axis orthogonal to the gravity axis and passing through the center of the cavity. The vertical sidewalls are suddenly raised and lowered to uniform temperatures  $T_h = T_0 + \Delta T/2$  and  $T_c = T_0 - \Delta T/2$ , while the top and bottom walls are thermally insulated. Thus, the air flow inside the cavity is simultaneously subjected to thermal and rotational buoyancy forces. By assuming the Boussinesq approximation in both the body and centrifugal forces, the thermal and the rotational buoyancies with the Coriolis forces acting on the flow are, respectively, equal to  $\rho_0 g \alpha (T - T_0)$ ,  $-\rho_0 \alpha (T - T_0) \Omega \times r$  and  $-2\rho_0 \Omega \times V$ .

**Governing equations in 2-D Cartesian coordinates in**



**Dimensional form:** The governing equations of mass,

momentum and energy can be expressed for two dimensional flows with orthogonal rotation axis in rotating coordinate free form as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{Du}{Dt} = -\frac{1}{\rho_0} \frac{\partial p_m}{\partial x} - \alpha(T - T_0)g \sin \Omega t + v \nabla^2 u + 2\Omega v - \Omega^2 \alpha(T - T_0)x$$

$$\frac{Dv}{Dt} = -\frac{1}{\rho_0} \frac{\partial p_m}{\partial y} + \alpha(T - T_0)g \cos \Omega t + v \nabla^2 v - 2\Omega u - \Omega^2 \alpha(T - T_0)y$$

Where

$$\frac{\partial p_m}{\partial x} = -\frac{\partial p}{\partial x} + \rho_0 \Omega^2 x + \rho_0 g \sin \Omega t$$

$$\frac{\partial p_m}{\partial y} = -\frac{\partial p}{\partial y} + \rho_0 \Omega^2 y - \rho_0 g \cos \Omega t$$

$$\frac{DT}{Dt} = k \nabla^2 T$$

**Scales for converting the Governing equation from dimensional form to dimensionless form:** The following scales have been considered for non-dimensionalization of the governing equations:

Length scale = H

$$\text{time scale} = \frac{H^2}{k}$$

$$\text{Velocity scale } V_{ref} = \frac{k}{H}$$

$$\text{Temperature } \theta = \frac{T - T_c}{T_h - T_c}$$

$$\text{Pressure} = \frac{P_m}{\rho V_{ref}^2}$$

**Dimensionless Parameters:** Following dimensionless parameters are obtained after non-dimensionalization of the governing equations:

AR = Aspect ratio

Ra = Rayleigh number

Ta = Taylor number

Raw = Rotational Rayleigh number

Pr = Prandtl number

## Result & Analysis

The simulation of convection has been done by varying the dimensionless parameter such as Rayleigh number, Rotational Rayleigh Number and Taylor Number in order to find the effect on convection phenomenon.

### 1. Computational results at Ra = 10<sup>5</sup>, Ta = 2 & Ra<sub>w</sub> = 1

The flow is mainly buoyancy driven flow as depicted in fig.1 having a single primary roll while fig.2 shows the corresponding isotherms. The effect of the insulated wall is to push the circulating fluid inwards resulting in slight distortion of isotherms near the walls while stable thermal stratification prevails in the core of the cavity. The stream function plot shows initially two secondary rolls, emanating from a common core, near the heated and cold walls, respectively.

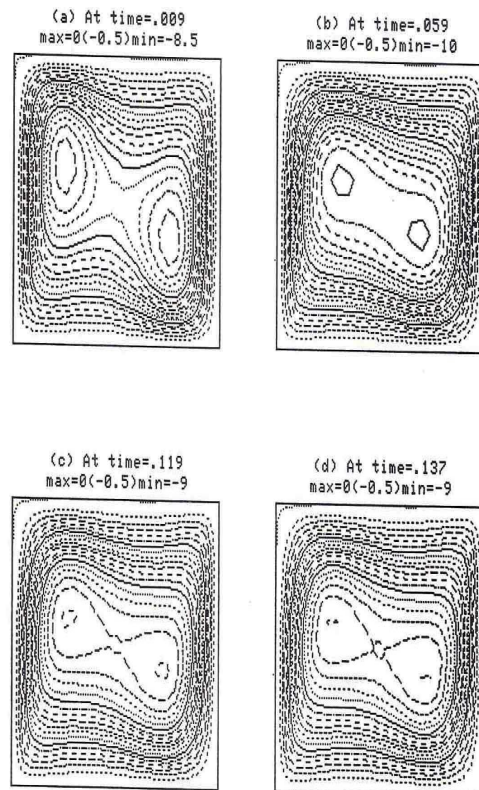


Fig 1.: streamline contours at different time instants at Ra = 10<sup>5</sup>, Ta = 2 & Ra<sub>w</sub> = 1

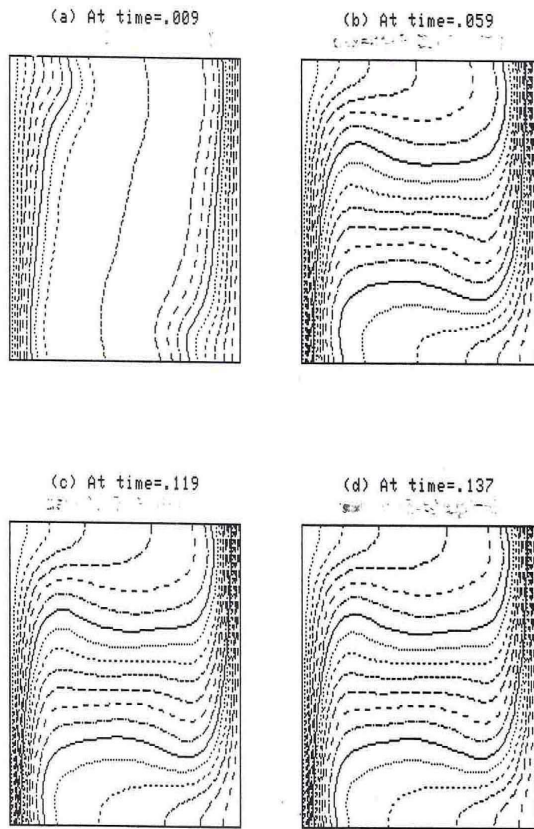


Fig . 2: Isotherm contours at different time instants at  $Ra = 10^5$ ,  $Ta = 2$  &  $Ra_w = 1$

2. **Computational results at  $Ra = 10^5$ ,  $Ta = 2 * 10^4$  &  $Ra_w = 10^4$ :** The streamline plot & isotherm plot in fig. 3 and 4 initially depict the flow to be dominated by thermal buoyancy with two secondary rolls at the corners. With the passage of time these rolls merge into a single roll rotating in direction to the previously obtained roll signifying the dominance of rotation. These rolls, due to periodic dominance of thermal and Coriolis forces over each other, get transformed in two secondary rolls at the corners which ultimately merge into a single roll.

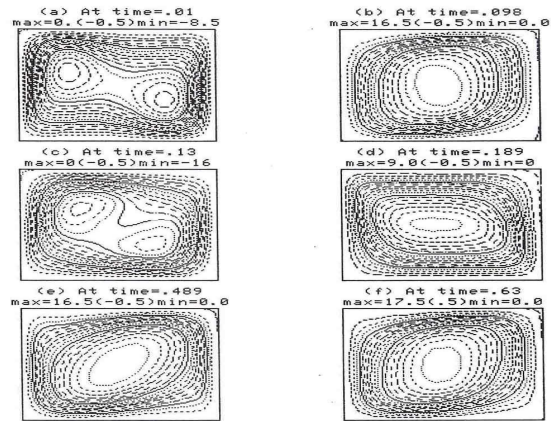


Fig . 3: streamline contours at different time instants at  $Ra = 10^5$ ,  $Ta = 2 * 10^4$  &  $Ra_w = 1$

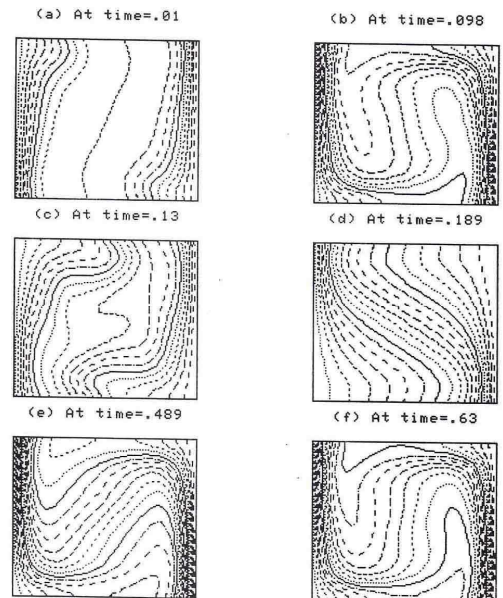


Fig . 4: Isotherm contours at different time instants at  $Ra = 10^5$ ,  $Ta = 2 * 10^4$  &  $Ra_w = 1$

### Conclusion and Future Work

The thesis work can be concluded as follows:

1. A single primary role is obtained when the flow is dominated by a single force and secondary rolls are obtained when there is partial domination of other force over the flow driven by single force.
2. Two horizontally aligned rolls rotating in opposite direction are obtained when the flow is

equally dominated by two forces (especially at higher  $Ta$ ).

3. Thin boundary layer and stable stratification in the core is obtained when the flow is dominated by thermal buoyancy.
4. A mushroom shaped plume is obtained when the flow is dominated by rotational forces. It signifies heat transfer in the core of the cavity.
5. The future scope of works is that we can simulate the problem by taking forced flow.

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